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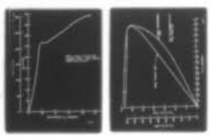

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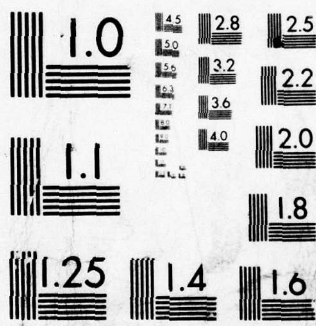
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THE CONSTANT VOLUME HYPOTHESIS FOR THE INELASTIC DEFORMATION OF METALS IN THE SMALL STRAIN RANGE

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P. Hewelt and E. Krempl

ABSTRACT

In plasticity and creep theory it is generally assumed that inelastic deformations are volume preserving. Available tensile and creep tests were evaluated to see whether the experiments confirm the constant volume assumption. None of the experiments which were done on a variety of structural metals confirmed the constant volume hypothesis.

THE CONSTANT VOLUME HYPOTHESIS FOR THE INELASTIC DEFORMATION
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Introduction

In the plasticity and creep theories for infinitesimal deformation the plastic strains and the creep strains, respectively, are usually introduced as deviators* and the total strain is split additively into elastic, plastic and creep parts

$$d\epsilon_{ij} = d\epsilon_{ij}^{el} + d\epsilon_{ij}^{pl} + d\epsilon_{ij}^{cr} \tag{1}$$

so that

$$d\epsilon_{ii} = d\epsilon_{ii}^{el} \tag{2}$$

i.e., the classical theories predict that for every stress state the mean normal strain is elastic and that the mean normal strain and pressure are related by the elastic bulk modulus. Usually the above results are motivated by the statement, plastic and creep deformations are isochoric and this is referred to as the constant volume hypothesis.

In the following we evaluate published experimental results to see whether the constant volume hypothesis is verified. We mention that the results are not from the "classical" plasticity or creep literature.

* Other constructions are of course possible.

Check on the Constant Volume Hypothesis

a) Tensile Test*

A simple tensile test affords the possibility of checking on the validity of (1) if the applied stress, the axial strain and the two perpendicular transverse strains are measured simultaneously. If the material is truly isotropic the measurement of one transverse strain is sufficient. If (1) is a realistic representation of metal behavior the graph of $\sigma_{kk} = \sigma$ vs $\epsilon_{ii} = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$ determined from a tensile test should be a straight line [1,2].

A literature survey was made and the results are summarized in Table 1. All tests are done at room temperature. In none of the cases was a linear relationship of σ_{kk} vs ϵ_{kk} observed, rather the two types of curves shown in Figs.1 and 2, respectively appeared as typical results. In some of the investigations [3,4,5,6] permanent volume changes were determined. With the exception of [3] where a decrease in volume was found, a permanent volume increase was reported. This permanent volume change depended on the maximum strain and was generally small ($\approx 2 \times 10^{-4}$ for a true plastic strain of 4%, [5] Fig.9) but measurable.

Measurements of Poisson's ratio ν during cyclic testing of tensile test specimens under completely reversed strain control [9,10] show clearly that the usually assumed value of $\nu = 1/2$ is not attained at a total strain range of 2% ([9], Figs.7,8,9) and at a plastic strain five times the elastic strain ([10], Fig.8). Moreover, the graphs in [9] and [10] reveal a gradual transition from the elastic value of Poisson's ratio to $\nu = 1/2$. These tests then do not support the usual assumption of $\nu = 1/2$ for plastic strains. It

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* We are only considering test data in small strain range.

should be noted that a total of four different steels were investigated in [9] and [10].

b) Creep Tests

No elevated temperature creep tests were found during which the axial and the transverse strains were measured simultaneously. However, room temperature creep tests during which the three perpendicular strains were measured by means of strain gages on 1100 Al ([11], Figs.12 and 13) show clearly that volume is not preserved during creep, rather ϵ_{ii}^{cr} increases continuously with time, see [12], Fig.3. The increase is considerable and cannot be neglected.

Discussion

The foregoing literature survey showed very clearly that the constant volume assumption usually employed in creep and plasticity theory is not supported by the experiments. Supporters of the constant volume hypothesis could point to the fact that isotropy had to be invoked in most of the tests listed in Table 1 to arrive at our conclusion. While the assumption of isotropy was necessary in most cases, there are other instances listed in Table 1 where strain was measured in three perpendicular directions. In addition the permanent volume changes were determined by other than strain measurements. Since they are shown to be consistent with the strain measurements [4,5] the volume changes obtained from the strain measurements are reliable. Indeed, no direct experimental support for the constant-volume hypothesis could be found and most plasticity textbooks take the constant volume hypothesis for granted. An exception is [13]. The experimental evidence cited in [13], however, pertains to large prior deformations (16% and higher reductions in diameter). The experiments show that volume increased permanently by an amount that is

TABLE 1

SURVEY OF PREVIOUS WORK ON σ_{kk} vs ϵ_{kk} IN A TENSILE TEST

Ref.	Material	Isotropy Assumed	σ_{kk} vs ϵ_{kk} is Straight Line	Remarks
[7]	Al 24ST sheet, flat specimen Fully killed low carbon steel, circular specimen	Yes Yes	No No	All σ_{kk} vs ϵ_{kk} curves have the shape of Fig.1. Max. axial strain ≈10%.
[8]	24S-T4 AL rolled 14S-T6 AL extruded 75S-T6 AL extruded	No	No	σ_{kk} vs ϵ_{kk} curves have shape of Fig.2
[3]	Six "high quality steels" Carbon steels Anneal 625°C, 1½ hrs in protective atmosphere	Yes	No	Sharp yield point, σ_{kk} vs ϵ_{kk} shape of Fig.2; permanent volume decrease .1 to 2% observed after testing. Maximum axial strains ≈ 3%.
[1,2]	AL 1100 as received AL 1100, annealed 600°F, 2 hrs Cu as received Cu, annealed 750°F, 2 hrs Low carbon steel, annealed 1250°F, 1 hr	Yes	No	σ_{kk} vs ϵ_{kk} curves have the shape of Fig.1 except for carbon steel where both the shape of Figs.1 and 2 is observed.
[4]	AISI 4330 } with special AISI 4310 } heat treatment	No*	No	3% maximum strain, unloading and reloading experiments; permanent volume increase; σ_{kk} vs ϵ_{kk} curves have the shape of Fig.1
[5]	HY 80 } special MARAGING steel } heat treatment	No*	No	3% maximum strain, unloading and reloading experiments; permanent volume increase; σ_{kk} vs ϵ_{kk} curves have the shape of Fig.1

* Readings from four 90° strain gage rosettes; we infer that isotropy was not assumed in [4] and [5].

small compared to the total deformation at the first measured point. From these measurements very little can be inferred about the behavior in the small strain range of interest here.

We further see that the detailed nonlinear behavior of the volume change with deformation is still in doubt (compare Figs.1 and 2) and may depend on the material. To this end additional experiments are necessary.

Behavior of Metals under Hydrostatic Pressure

Another experimental observation related to volume change is the behavior of metals under hydrostatic pressure. Available experimental evidence [14] and the results cited in [3] (specifically Ref.[1] in [3]) support a linear elastic relation between hydrostatic pressure and volume change*. In many instances this linear relation under hydrostatic stress is thought to imply (2). However, this is not so. It rather requires

$$\text{and} \qquad \left.\begin{aligned} d\epsilon_{ii}^{pl} &= 0 \\ d\epsilon_{ii}^{cr} &= 0 \end{aligned}\right\} \quad \text{for } \sigma_{ij} = p\delta_{ij} \tag{3}$$

i.e., the first invariants of plastic and creep strain increments must be zero if the stress state is hydrostatic. In the above σ_{ij} denotes the stress tensor, p the pressure and δ_{ij} is the Kronecker delta. Equation (2) implies that (3) holds for every stress state not just the one given in (3). It is therefore more restrictive than required by the test results under hydrostatic stress.

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* In this study we are only interested in pressure levels comparable in magnitude to stresses encountered in stress analysis. The ultrahigh pressures to which the data of [14] extend are of no interest here.

It appears therefore that the constant volume hypothesis is not reproduced by the recent experiments quoted herein. Since they cover a wide variety of metals there is reason to believe that the inelastic deformation of metals may not be volume preserving. In addition the constant volume assumption requires special precautions in computational applications [15] which could conceivably be avoided if an improved constitutive model would be available. It is therefore urgent to develop faithful constitutive equations for metals reflecting the recent observations which are obtained with much better equipment than was available at the time of the development of the "classical theories" of creep and plasticity.

Acknowledgement

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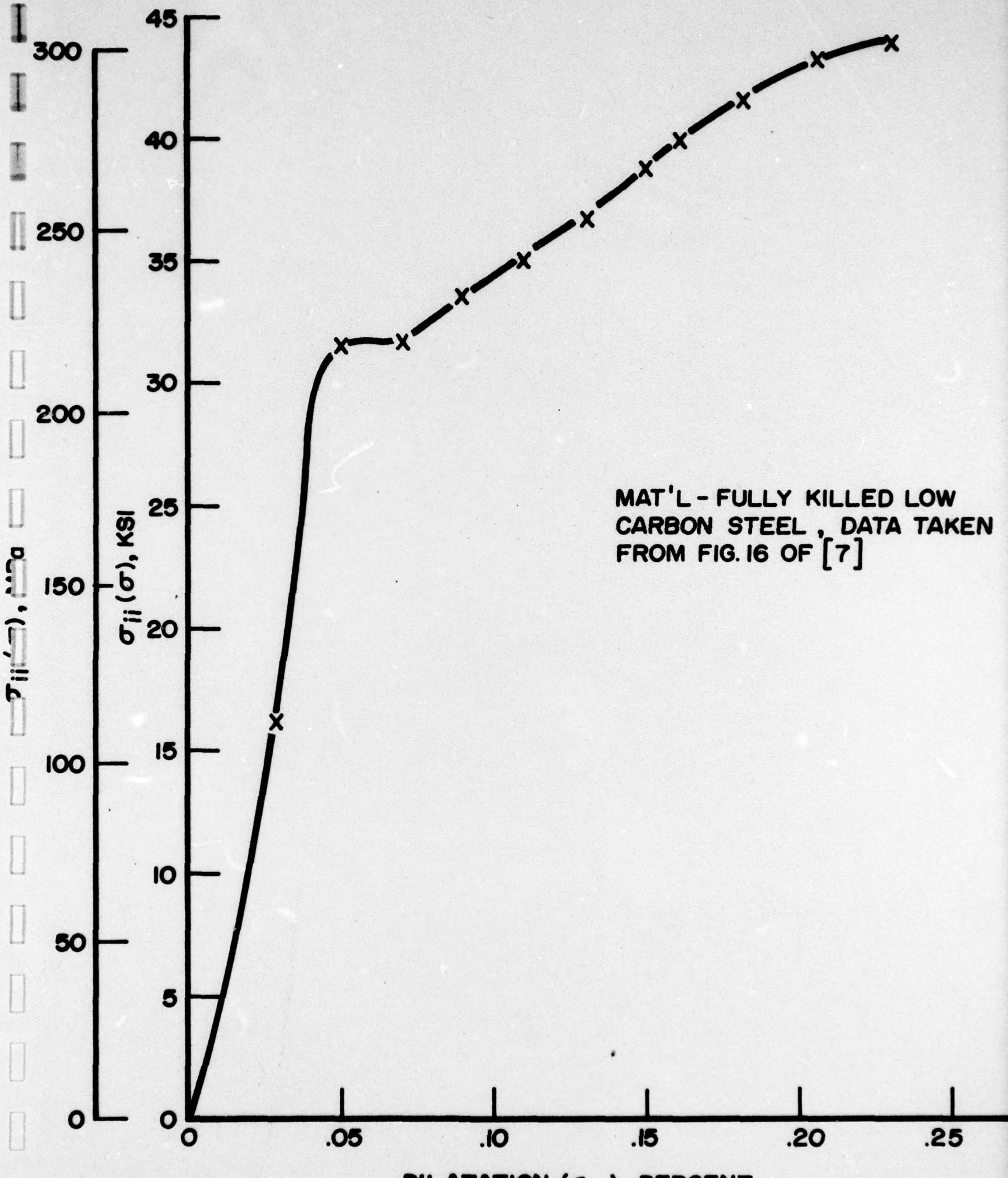
FIGURE CAPTIONS

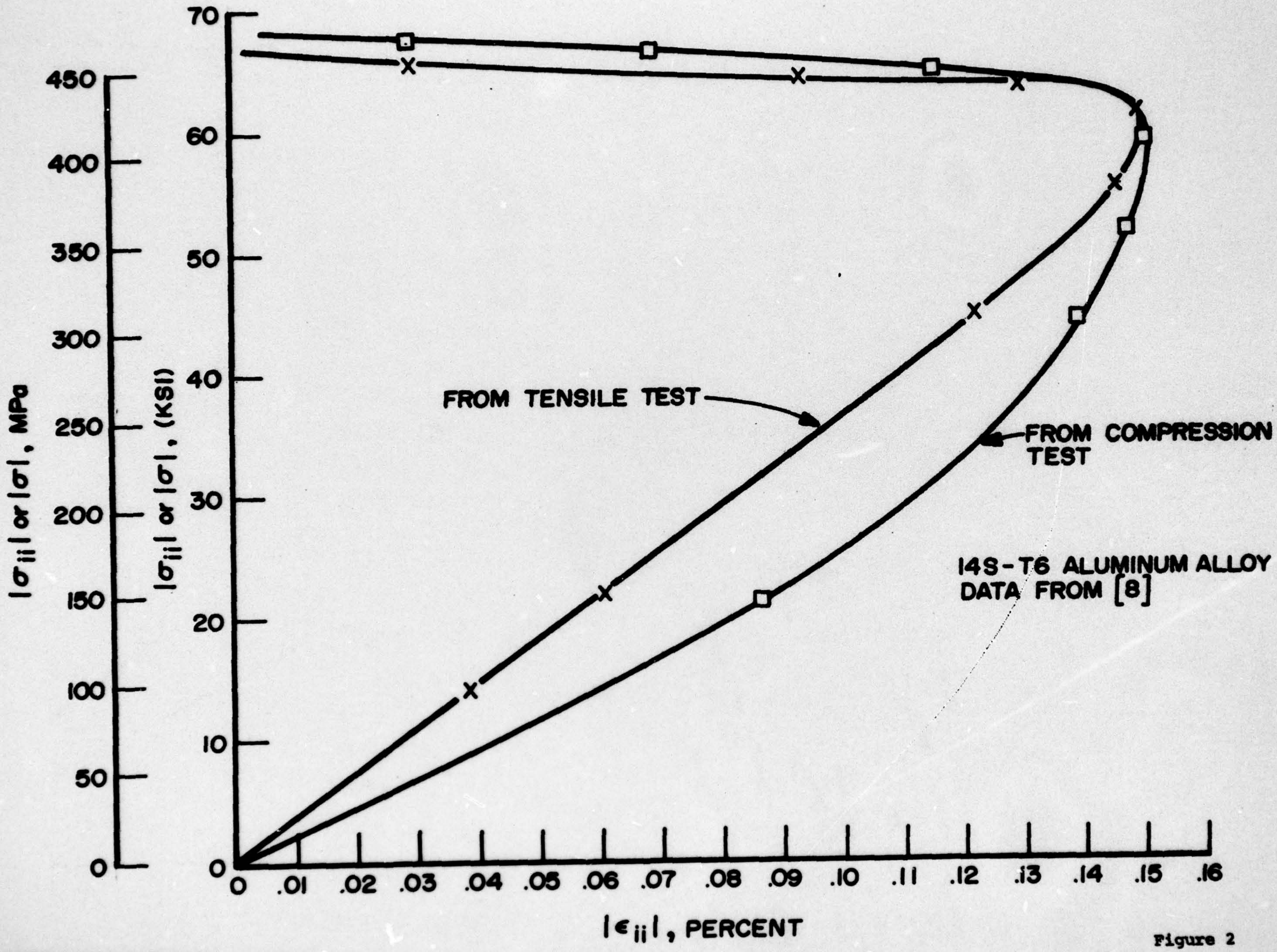
Figure 1 Axial Stress σ (or Mean Normal Stress σ_{ii}) vs. Measured Volume Change ϵ_{ii} as Determined from the Results of Tensile Tests in [7].

Figure 2 Absolute Value of Axial Stress σ (or Absolute Mean Normal Stress $|\sigma_{ii}|$) vs. Measured Absolute Volume Change $|\epsilon_{ii}|$ as Determined from the Results of Tensile and Compression Tests Reported in [8].



300
250
200
150
100
50
0
45
40
35
30
25
20
15
10
5
0
$\sigma_{ii}(\sigma)$, KSI
0
.05
.10
.15
.20
.25
DILATATION (ϵ_{ii}), PERCENT
MAT'L - FULLY KILLED LOW CARBON STEEL, DATA TAKEN FROM FIG. 16 OF [7]

Figure 1



FROM TENSILE TEST
FROM COMPRESSION TEST
14S-T6 ALUMINUM ALLOY
DATA FROM [8]
$|\sigma_{ii}|$ or $|\sigma|$, MPa
$|\sigma_{ii}|$ or $|\sigma|$, (KSI)
$|\epsilon_{ii}|$, PERCENT

Figure 2